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Algorithms & Data Structures — Homework 7 — HS 18

Exercise Class (Room & TA): _____

Submitted by: _____

Peer Feedback by: _____

Points: _____

Exercise 7.1 *Heapsort.*

Given the array [H,E,A,P,S,O,R,T,I,N,G], we want to sort it in ascending alphabetical order using Heapsort.

1. From the lecture, you know a method to construct a heap in linear time. Draw the resulting max binary heap if this method is applied to the above array.
2. Sort the above array in ascending alphabetical order with heapsort, beginning with the heap that you obtained in 7.1.1. Draw the array after each intermediate step in which a key is moved to its final position.
3. You are given a max binary heap with n pairwise distinct elements, stored in an array. Specify the set of array indices where the minimum element can be found. Provide a proof.

Exercise 7.2 *Quicksort* **(1 Point for 7.2.2)**.

1. Given the array [Q, U, I, C, K, S, O, R, T, I, N, G], we want to sort it in ascending alphabetical order using Quicksort. When partially sorting a subarray (with procedure Aufteilen), always use the last element as the pivot. Draw the array after each intermediate step in which a key is moved to its final position.
2. Prove by induction on n that Quicksort (as seen in the lecture and described in the script) performs at most $2n^2$ comparisons of keys when sorting an array of n pairwise distinct numbers, where $n \geq 0$ is a non-negative integer.
3. You and your friend have a competition to see who can implement the fastest sorting algorithm. You choose to implement Heapsort, and your friend chooses to implement Quicksort. By peaking at her code, you noticed that your friend's implementation of Quicksort always chooses the element at index $\lfloor \frac{k}{2} \rfloor$ as the pivot when sorting a subarray of length k. (That is, the additional very first step of Aufteilen is to swap the elements in positions $\lfloor \frac{k}{2} \rfloor$ and r, where r is the last index of the subarray.)

   Given an array with elements $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, what order maximizes the number of comparisions that your friend's Quicksort algorithm must perform? Give a solution for how to arrange the elements of an array of length n, so that you will always win the competition when n is large enough.

Exercise 7.3 *Resistors* **(1 Point)**.

When assembling electronic circuits, it is important to make sure that the used resistors have exactly the right values in order to guarantee the correct functionality of the circuit. Since one does not always have all possible resistor values, one often uses a series connection of multiple resistors to achieve a certain total resistance. For example, one can produce a total resistance $R = 27\Omega$ by the two resistors $R_1 = 12\Omega$ and $R_2 = 15\Omega$ connected in series.

Develop an algorithm that decides for a given box full of n resistors, whether it is possible to achieve a certain resistance R by a series connection of at most 2 resistors. The algorithm should be as fast as possible, but only use concepts that you have learned in this class. Express the worst-case complexity of the algorithm in $\mathcal{O}$ notation.

Exercise 7.4 *Coin Collecting* **(1 Point)**.

You are an avid coin collector and you have a huge bucket of coins in front of you. You are interested in getting the oldest coins out of the bucket to put into a display case.

There are n coins in your bucket, and you want to place the m oldest coins into your display case, where m is much smaller than n. Develop an algorithm to find the m oldest coins. The algorithm should be as fast as possible (in the worst case, as always), but only use concepts that you have learned in this class. Express the worst-case complexity of the algorithm in $\mathcal{O}$ notation.

Submission: On Monday, 12.11.2018, hand in your solution to your TA *before* the exercise class starts.